

Find the absolute extrema for the given function on the closed interval.

1. $f(x) = x^3 - 6x^2; [-1, 3]$

The table below shows values of f and f' . The function is twice differentiable on the interval $[0, 6]$.

| | | | | | | | |
|---------|---|----|----|---|---|---|----|
| x | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| $f(x)$ | 2 | 5 | -1 | 7 | 3 | 2 | 0 |
| $f'(x)$ | 1 | -2 | 0 | 1 | 5 | 8 | 12 |

2. Is there an x value where $f(x) = 3$ in the interval $0 \leq x \leq 1$? Justify your answer.

3. Is there an x value where $f'(x) = -4$ in the interval $3 \leq x \leq 4$? Justify your answer.

4. Is there an x value where $f'(x) = 9$ in the interval $4 \leq x \leq 6$? Justify your answer.

5. Write the equation of the line tangent to f at $x = 4$. Use the equation of the tangent line to approximate $f(4.1)$.

6. Evaluate: $\lim_{x \rightarrow 1} \frac{f(2x)+x}{x^2+2x-3}$.

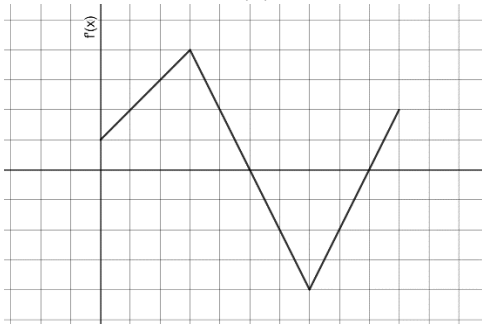
7. If $h(x) = \frac{f(3x)}{x^2+2}$. Find $h'(1)$.

8. If $h(x) = f(g(x))$ and $g(x) = x^3 + x - 4$, find $h'(2)$.

9. Estimate $f'(2.5)$.

For questions 10-12, find each of the following: Increasing and decreasing intervals, local extrema, concave up and concave down intervals, and points of inflections.

10. The graph of $f'(x)$ is given below.



11. $y = x^4 - 18x^2$

12. $f(x) = xe^{2x}$

Answer each question in a complete sentence.

13. Air is being pumped into a spherical balloon at a rate of $5 \text{ cm}^3/\text{min}$. Determine the rate at which the radius of the balloon is increasing when the diameter of the balloon is 20 cm .

14. A 15-foot ladder is resting against the wall. The bottom is initially 10 feet away from the wall and is being pushed towards the wall at a rate of $\frac{1}{4} \text{ ft}/\text{sec}$. How fast is the top of the ladder moving up the wall 12 seconds after we start pushing?

15. A tank of water in the shape of a cone is leaking water at a constant rate of $2 \text{ ft}^3/\text{hour}$. The radius of the tank is 5 feet and the height of the tank is 14 feet. At what rate is the depth of the water in the tank changing when the depth of the water is 6 feet?

16. Answer the following questions about the function $h(x)$ given the following information:

| x | 0 | $0 < x < 1$ | 1 | $1 < x < 2$ | 2 | $2 < x < 3$ | 3 |
|----------|----|-------------|---|-------------|-----|-------------|----|
| $h(x)$ | 2 | + | 0 | - | -3 | - | 0 |
| $h'(x)$ | -3 | - | 0 | - | DNE | + | 2 |
| $h''(x)$ | 1 | + | 0 | - | DNE | - | -2 |

a. When is the function $h(x)$ increasing?

b. Where does $h(x)$ have a maximum? If there is no maximum, explain why.

c. Where does $h(x)$ have a point of inflection? If there is no point of inflection, explain why.

d. Graph $h(x)$.

