

Simplify each expression.

a) $\sin x(\cot x + \tan x)$

b) $\tan^2 x \cos^2 x + \cot^2 x \sin^2 x$

c) $\frac{1 + \sin \theta}{\cos \theta} + \frac{\cos \theta}{1 + \sin \theta}$

d) $\sec^2 y - \cot^2\left(\frac{\pi}{2} - y\right)$

e) $\frac{\sin x + \tan x}{1 + \sec x}$

f) $\sec x - \tan x \sin x$

g) $\frac{\sec x}{\cos x} - \frac{\tan x}{\cot x}$

Prove each identity.

a) $2 - \sec^2 z = 1 - \tan^2 z$

b) $\sec x - \cos x = \sin x \tan x$

c) $\sec^2 x \tan^2 x + \sec^2 x = \sec^4 x$

d) $\cot \beta + \tan \beta = \frac{\sec^2 \beta}{\tan \beta}$

e) $\cos^2 x - \sin^2 x = 2 \cos^2 x - 1$

f) $\frac{\cos x - \cos y}{\sin x + \sin y} + \frac{\sin x - \sin y}{\cos x + \cos y} = 0$

g) $\frac{\cos\left(x - \frac{\pi}{2}\right)}{\sin\left(x - \frac{\pi}{2}\right)} = -\tan x$

h) $(1 + \sin y)(1 + \sin(-y)) = \cos^2 y$

i) $\frac{\sec \beta - 1}{1 - \cos \beta} = \sec \beta$

j) $\frac{1 + \cos x}{\sin x} = \csc x + \cot x$

k) $(\sin x + \cos x)(\tan x + \cot x) = \sec x + \csc x$

l) $\frac{\cos \phi}{1 - \sin \phi} = \frac{1 + \sin \phi}{\cos \phi}$

Solve each equation over $[0, 2\pi)$.

a) $\sin^2 x - 6 \sin x + 5 = 0$

b) $2 \cos^2 x - 5 \cos x + 2 = 0$

c) $2 \cos^2 x = -\cos x$

d) $\csc^2 x - \csc x - 2 = 0$

e) $4 \cos^2 x - 3 = 0$

f) $\tan^2 x + \tan x = 0$

g) $\sec^2 x - 2 \tan x = 4$

h) $(\cos x + 1)^2 = \sin^2 x$