

Evaluate each limit algebraically.

1) $\lim_{x \rightarrow \infty} \frac{8 - \frac{2}{x^2}}{\frac{7}{x^3} - 4}$

2) $\lim_{x \rightarrow \infty} \left(\frac{2}{x} + 5 \right) \left(\frac{3x^2 - 1}{x^2} \right)$

3) $\lim_{x \rightarrow -\infty} \frac{3 - 4x - x^2}{x + 1}$

4) $\lim_{x \rightarrow \infty} \frac{6x^2 - 9}{x^3 - 12x + 3}$

5) $\lim_{x \rightarrow \infty} \frac{\sqrt{2x^2 - 5}}{4x - 1}$

6) $\lim_{x \rightarrow \infty} \frac{x^{100}}{e^{5x}}$

7) $\lim_{x \rightarrow \infty} \frac{x^{100}}{\ln 6x}$

8) $\lim_{x \rightarrow \infty} \frac{(2-x)(4x+4)}{(2x+1)(2+3x)}$

9) $\lim_{x \rightarrow \infty} \frac{8x - 2}{5 - 4x}$

10) When $g(x) = \begin{cases} \frac{3x}{x+1} & x \leq 0 \\ \frac{1}{x^2} & x > 0 \end{cases}$, find each limit:

$$\lim_{x \rightarrow \infty} g(x)$$

$$\lim_{x \rightarrow -\infty} g(x)$$

$$\lim_{x \rightarrow 0^-} g(x)$$

$$\lim_{x \rightarrow 0^+} g(x)$$

Find all horizontal and vertical asymptotes.

$$11) f(x) = \frac{1}{x-3}$$

$$12) f(x) = \frac{1-x}{2x^2-5x-3}$$

13) Sketch a function that satisfies the stated conditions. Include any asymptotes.

$$\lim_{x \rightarrow 2} f(x) = -1 \quad \lim_{x \rightarrow \infty} f(x) = \infty$$

$$\lim_{x \rightarrow 4^-} f(x) = \infty \quad \lim_{x \rightarrow -\infty} f(x) = 2$$

$$\lim_{x \rightarrow 4^+} f(x) = -\infty$$

14) Prove that the function f is continuous at $x = 1$. $f(x) = \begin{cases} x^2 + 5, & x > 1 \\ -x + 7, & x \leq 1 \end{cases}$

15) Explain why there must be at least one value, for $-1 < x < 8$, where $g(x) = 1$.

x	-1	2	7	8
$g(x)$	5	4	-3	-5