Estimating Area and Definite Integrals NAME____

1. Water into being pumped out of a tank. The rate at which of the water has been pumped out is given by $R(t)$, where R is measured in ounces/minutes, and t is measures in minutes.

t	0	20	60	80	80	100	120
$R(t)$	1.2	2.8	4.0	4.7	5.1	5.2	4.8

   a. Estimate $\int_0^{120} R(t)\,dt$ using a left Riemann Sum with six subintervals.
   b. Estimate $\int_0^{120} R(t)\,dt$ using a right Riemann Sum with six subintervals.
   c. Estimate $\int_0^{120} R(t)\,dt$ using a trapezoidal approximation with six subintervals.
   d. Describe each of the following in the context of the problem:
      i. $R(70)$
      ii. $R'(70)$
      iii. $\int_0^{120} R(t)\,dt$
      iv. $\frac{1}{120}\int_0^{120} R(t)\,dt$

2. A particle moves along the x-axis with the velocity of $v(t)$ given by the table below. $v(t)$ is measures in meters/second, and t is measures in seconds.

t	0	40	70	90	100
$v(t)$	10	-3	4	9	-2

   a. Estimate $\int_0^{100} v(t)\,dt$ using a left Riemann Sum with four subintervals. Interpret in the meaning in the context of the problem.
   b. Estimate $\int_0^{100} |v(t)|\,dt$ using a right Riemann Sum with four subintervals. Interpret in the meaning in the context of the problem.
   c. Estimate $\frac{1}{100}\int_0^{100} v(t)\,dt$ using a trapezoidal approximation with four subintervals. Interpret in the meaning in the context of the problem.
   d. Estimate the acceleration of the particle at $t = 80$.

3. Estimate $\int_0^4 4 - x^2\,dx$ using
   a. Left Riemann Sum with 4 subintervals
   b. Right Riemann Sum with 4 subintervals
   c. Midpoint Riemann Sum with 4 subintervals
   d. Trapezoidal Approximation with 4 subintervals

4. Graph each integrand and use the areas to evaluate each integral.

   a. $\int_{-2}^{4} \frac{x}{2} + 3\,dx$
   b. $\int_{0}^{4} -2x + 4\,dx$
   c. $\int_{-3}^{3} \sqrt{9 - x^2}\,dx$
   d. $\int_{-2}^{1} |x|\,dx$

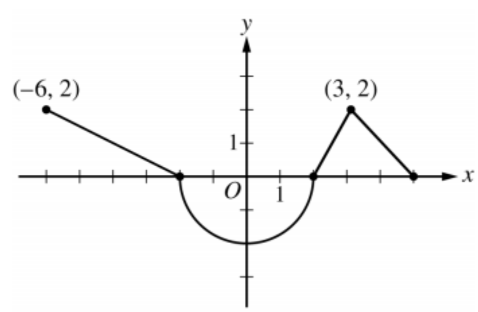
5. Suppose that f and g are continuous functions and that $\int_{1}^{2} f(x)\,dx = -4$, $\int_{1}^{5} f(x)\,dx = 6$, $\int_{1}^{5} g(x)\,dx = 8$.

   a. $\int_{2}^{2} g(x)\,dx =$
   b. $\int_{5}^{1} g(x)\,dx =$
   c. $\int_{1}^{2} f(x)\,dx =$
   d. $\int_{2}^{5} f(x)\,dx =$
   e. $\int_{1}^{5} [f(x) - g(x)]\,dx =$
   f. $\int_{1}^{5} [4f(x) - g(x)]\,dx =$

6. Suppose that f is continuous and that $\int_{0}^{3} f(t)\,dt = 3$ and $\int_{0}^{4} f(t)\,dt = 7$.

   a. $\int_{3}^{4} f(t)\,dt =$
   b. $\int_{4}^{3} f(t)\,dt =$

7. The graph of $f'(t)$ is given to the right. It consists of a semicircle and three line segments.
   The function f is defined as $f(x) = \int_{0}^{x} f'(t)\,dt$.
   Evaluate the following:



   a. $f(2) =$
   b. $f(3) =$
   c. $f(5) =$
   d. $f(-2) =$
   e. $f(-6) =$